M.Sc. - I (Mathematics) (New CBCS Pattern) Semester-I

PSCMTH05(A) - Numerical Analysis

P. Pages : 2 **GUG/S/25/13741**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT-I

1. a) Discuss the secant method and prove convergence of x_n to α under suitable condition. **10**

b) Assume $f(x), f'(x)$ and $f''(x)$ are continuous for all x in some neighbourhood of α and assume $f(\alpha)=0, f'(\alpha)\neq 0$, then prove that if x_0 is chosen sufficiently close to α the iterates $x_n, n\geq 0$ of $x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$, will converges to α. **10**

OR

c) Let $g(x)$ be continuous on $[a,b]$, and assume $g([a,b])\subset[a,b]$. Furthermore, assume there is a constant $0<\lambda<1$, with **10**

$$|g(x)-g(y)|\leq\lambda|x-y| \text{ for all } x,y\in[a,b]$$

Then $x=g(x)$ has a unique solution α in $[a,b]$. Also, the iterates $x_n=g(x_{n-1}),\ n\geq 0$ will converge to α for any choice of x_0 in $[a,b]$.

d) Discuss the comparison of Newton's method and secant method. **10**

UNIT-II

2. a) Let $x_1, x_2, ..., x_n$ be distinct real numbers, and let f be a given real valued function with $n+1$ continuous derivatives on the interval $I_t=H\{x_0, x_1, ..., x_n\}$, with t some given real number. Then prove that there exists $\xi\in I_t$ with **10**

$$f(t)-\sum_{j=0}^{n} f(x_j)i_j(t)=\frac{(t-x_0)....(t-x_n)}{(n-1)!}f^{(n+1)}(\xi)$$

b) Find the Hermite interpolating polynomial for which **10**

$p(a)=f(a)$ $p'(a)=f'(a)$

$p(b)=f(b)$ $p'(b)=f'(b)$

OR

c) Prove that for $k\geq 0$ **10**

$$f[x_0, x_1, ..., x_k]=\frac{1}{k!h^k}\Delta^k f_0, \text{ where } f_0=f(x_0) \,\&\, f_i=f(x_i)$$

d) For any two functions f and g and for any two constants α and β, show that **10**

$$\Delta^r[\alpha f(x)+\beta g(x)]=\alpha\Delta^r f(x)+\beta\Delta^r g(x) \quad r\geq 0$$

UNIT-III

3. a) Obtain an minimax polynomial approximation $a_1{}^*(x)$ for the function $f(x) = e^x$ on the interval $[1, -1]$. **10**

b) Assuming $[a, b]$ is finite, then prove that **10**

$$\lim_{n\to\infty} \|f - r_n{}^*\|_2 = 0$$

OR

c) Let $\{\phi_n(x)\} \mid n \geq 0\}$ be an orthogonal family of polynomials on (a, b) with weight function $w(x)$. With such a family we always assume implicitly that degree $\phi_n = n \quad n \geq 0$. **10**

If $f(x)$ is a polynomial of degree m, then prove that

$$f(x) = \sum_{n=0}^{m} \frac{(f, \phi_n)}{(\phi_n, \phi_n)} \phi_n(x)$$

d) Discuss the Gram-Schmidt theorem. **10**

UNIT-IV

4. a) Obtain the composite trapezoidal rule with error. Find the expression for the asymptotic error. **10**

b) Obtain the expression for Peano-Kernel error formula. **10**

OR

c) Assume $[a, b]$ is finite. Then prove that the error in Gaussian quadrature, **10**

$$E_n(f) = \int_a^b w(x) f(x) dx \sum_{j=1}^{n} w_j f(x_j)$$

Satisfies

$$|E_n(f)| \leq 2\left[\int_a^b w(x) dx\right] \rho_{2n-1}(f) \quad n \geq 1$$

With

$\rho_{2n-1}(f)$ the minimax error from $\rho_n(f) = \underset{\deg(q) \leq n}{\text{Infimum}} \|f - q\|_\infty$

d) Obtain Simpson three-eights rule of integration. **10**

5. a) Show that the Newton's method for determining a reciprocal root of A has the form **5**

$$x_{n+1} = x_n(2 - Ax_n)$$

b) Obtain the expression for $p_1(x)$ and $p_2(x)$ by Lagrange's interpolation. **5**

c) For $f, g \in C[a, b]$. Then prove that **5**

$$\|f + g\|_2 \leq \|f\|_2 + \|g\|_2$$

d) Define: **5**

i) Asymptotic error estimate

ii) Degree of precision

****